Electronic monopole sum rule and helium burning

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An analysis has also been made for the two signs of muons separately. The ratio of the mean rate of total energy loss for positive to that of negative muons is $1{\cdot}01 \pm 0{\cdot}07$, $1{\cdot}04 \pm 0{\cdot}08$, $0{\cdot}92 \pm 0{\cdot}06$ and $0{\cdot}84 \pm 0{\cdot}10$ at mean momenta of 6·3, 8·6, 14·4 and 27·7 GeV/*c* respectively. None is significantly different from unity.

Comparison with the results of other workers can be brief because of the paucity of direct experiments. Buhler *et al.* (1964) determined muon ranges in lead for momenta up to 2·48 GeV/*c* in an accelerator experiment and found values within 2% of the Sternheimer predictions. At a higher momentum, 8 GeV/*c*, Backenstoss *et al.* (1963) have studied the passage of negative muons through magnetized iron, with particular reference to bremsstrahlung and knock-on electron production where the energy transfer is above 1·6 GeV. Agreement with expectation was found to within approximately 3%. Our own data are not inconsistent with the results of either of these two experiments.

In conclusion, there is no marked divergence from expectation for the mean total energy loss in iron for muons in the momentum range 5–40 GeV/*c*.

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Electric monopole sum rule and helium burning

Abstract. The possibility of helium burning occurring through the reaction $^{12}C(\alpha, e^{\pm})^{16}O$ is examined for a temperature of 10^8 K. As the isoscalar electric monopole sum rule does not rule this out, the results of an electron scattering experiment are reported which demonstrate the absence of electric monopole excitations in the continuum beyond the threshold for $^{12}C + {}^{4}He$.

A step in the evolution of stars (Burbidge *et al.* 1957, Salpeter 1957) leading to the build up of heavy elements, is helium burning through the reaction $^{12}C(\alpha, \gamma)^{16}O$.

Radiative alpha capture on carbon to form ^{16}O is thought to depend strongly on the $J^P = 1^-$, $(7{\cdot}115 \pm 0{\cdot}012)$ MeV level in ^{16}O, which is 40 keV below the $^{12}C + ^4He$ threshold. The γ ray width of this state has been measured (Swann and Metzger 1957) by resonance fluorescence, whilst the α width has been deduced (Loebenstein *et al.* 1967) only indirectly from the $^6Li(^{12}C, d)^{16}O^*$ reaction, with the result that the extracted, dimensionless reduced alpha width $\theta_\alpha{}^2$ lies in the range 0·06–0·14.

Attempts to measure directly (Larson and Spear 1964, Adams *et al.* 1968, Jaszczak *et al.* 1970) the capture cross section as a function of α particle energy E_α, are complicated by the prolific fast neutron background of the reaction $^{13}C(\alpha, n)^{16}O$, and of other contaminants. The p wave Coulomb barrier rapidly diminishes the cross section making it difficult to measure below $E_\alpha = 1{\cdot}6$ MeV.

Detection of the γ ray alone in any case eliminates one possible alternative channel through the reaction $^{12}C(\alpha, e^\pm)^{16}O$, which is s wave α particle capture followed by internal pair formation to de-excite the $^{16}O^*$. Since s wave capture leads to continuum states for $^{16}O^*$ with $J^P = O^+$, no ground state de-excitation γ rays can be emitted as the minimum angular momentum of a photon is unity. By internal pair formation decay takes place with a width given by (Oppenheimer and Schwinger 1939).

$$\Gamma_{E0} = \frac{\alpha^2}{135\pi} E^5 |M_{00}|^2. \tag{1}$$

Here α is the fine structure constant, E the energy of the transition, and M_{00} the electric monopole matrix element. Whether this process contributes significantly to helium burning or not depends on two factors, the temperature of the star and the magnitude of M_{00}. A temperature of $1{\cdot}2 \times 10^8$ K is found to maximize the burning rate when this is taken (Salpeter 1957) to be proportional to the product of a Breit-Wigner nuclear reaction cross section, and a Boltzmann factor, $\exp(-E\alpha/kT)$, for the α particle to have random thermal energy E_α. At $T = 1{\cdot}2 \times 10^8$ K, $E_\alpha = 0{\cdot}23$ MeV, and an s wave α particle has a Coulomb barrier penetrability fifty times greater than a p wave α particle at this E_α.

The monopole matrix element M_{00}, which is the expectation value of the operator $\Sigma r_p{}^2$ summed over all protons in the nucleus, is not known above the threshold. Below the threshold the first excited state of $^{16}O^*$ is at 6·051 MeV with $J^P = O^+$, and the monopole matrix element is 38 mb (millibarn), as determined by measurement (Devons and Goldring 1954) of Γ_{E0}, or by inelastic electron scattering (Bishop *et al.* 1964). This transition represents only 4% of the isoscalar E0 sum rule (Ferrell 1957), which is expressed by the relations

$$f_{n0} = \frac{2M\hbar\omega_{0n}}{\hbar^2 \langle r^2 \rangle} |M_{n0}|^2$$
$$\sum_n f_{n0} = A. \tag{2}$$

Here f_{n0} is the oscillator strength to $J^P = O^+$ states labelled by the index n, M is the nucleon mass, $\hbar\omega_{n0}$ is the nth state excitation energy, $\langle r^2 \rangle^{1/2}$ is the mean square radius of the nucleus, M_{n0} is the monopole matrix element of the nth state and A is the atomic number. The remaining 96% of this sum rule, if concentrated in the region of excitation energy between threshold ($Q = 7{\cdot}148$ MeV), and 8·55 MeV, (which corresponds to $E_\alpha = 1{\cdot}86$ MeV, the lowest bombarding energy at which the terrestrial alpha capture experiments have been performed (Jaszczak *et al.* 1970) could give a value of M_{00} almost as large as $24 \times 38 = 912$ mb. From equation (1) a decay width $\Gamma_{E0} = 1{\cdot}06$ meV would result which, combined with the factor of 50 from the relative Coulomb barrier penetrability, gives an effective $\Gamma_{E0} = 50{\cdot}3$ meV to compare with the $\Gamma_{E1} = 80$ meV (Swann and Metzger 1957) characterizing the electric dipole radiative capture.

The comparison is valid provided the reduced α particle width θ_α^2 is the same for both s- and p-wave α particles. The value of 0·06 of the Wigner–Teichmann limit mentioned above (Loebenstein *et al.* 1967) is quite large, and values ten times smaller are far more common. A direct measurement in the capture reaction would be desirable but is clearly difficult to make (Jaszczak *et al.* 1970). If θ_α^2 were as low as 0·006 then the pair production process might dominate since the effective Γ_{E0} would be 503 meV. On the other hand, θ_α^2 for s wave capture might itself be very small because the Pauli principle prohibits the addition of the four 1s nucleons of the α particle to ^{12}C, which already has four 1s nucleons. However the ground state of ^{12}C has a large 4 particle–4 hole component in its wavefunction which might restore θ_α^2 to typical values.

This problem is resolved by electron scattering experiments through which E0 transitions are easily excited (Bishop 1963). The high energy resolution now available (Hogg *et al.* 1971) allows the scattering to continuum states just above the threshold to be studied. Inelastic scattering with excitation (Bishop *et al.* 1964) of the levels at 6·923 MeV, ($J^P = 2^+$) and 7·115 MeV, ($J^P = 1^-$), dominates the spectrum and due to finite experimental resolution the radiative tail partly overlaps the region just above threshold. On figure 1 is plotted the inelastic electron spectrum obtained after

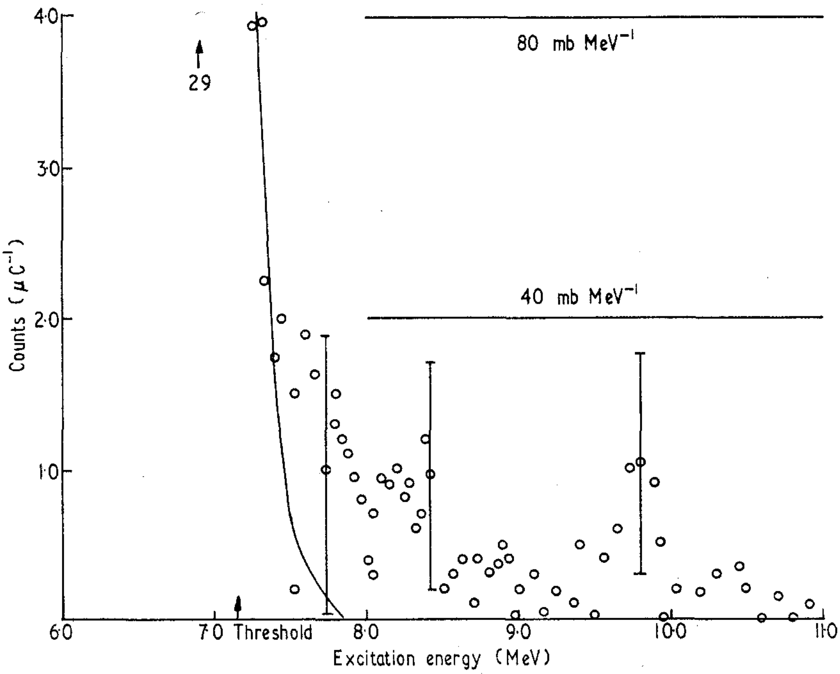

Figure 1. The inelastic spectrum of electrons scattered by ^{16}O in the region just above the threshold for disintegration into $^{12}C + ^4He$. Indicated are the trailing edge of the inelastic peak corresponding to excitation of levels at 6·92 MeV and 7·12 MeV, and the counting rates expected for two values of electric monopole matrix element.

subtraction of the radiative tails due to the elastic peak, and inelastic peaks from all lower levels (Bishop *et al.* 1964). The low energy side of the 6·923–7·115 MeV doublet scattering peak is indicated. Poor statistics are the result of subtraction of the dominating elastic radiation tail. Also plotted are the levels of count rate expected if the missing E0 oscillator strength were distributed uniformly as a line source of magnitude expressed in units of mb per MeV energy interval in the continuum. It is clear that no concentration of E0 strength exists sufficient to make the pair production process a large contributor to helium burning at temperatures of 10^8 K.

At higher stellar temperatures internal pair production might contribute. From equation (2) the monopole matrix element squared $|M_{n0}|^2$ diminishes in inverse proportion to the excitation energy $\hbar\omega_{0n}$. Thus the width Γ_{E0} from equation (1) increases proportionately to E^4 provided the E0 sum rule is concentrated in a narrow band of excitation energy. However, with increasing energy the Coulomb barrier penetrability tends to unity for all partial waves. It clearly would be of interest to find where the rest of the E0 oscillator strength is situated in excitation energy of ^{16}O.

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